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A STUDY OF THE ACTION OF WILD FIRES ON REMOTE FORESTS

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A stochastic model is developed to describe interaction of fires and vegetation in remote forest regions to demonstrate that, given certain circumstances, fires have a dominant effect on the long term, as well as the short term, forest structure. The model describes interactions peculiar to rainforest regions of Tasmania, Australia, but can be adapted to any situation where a similar dependence of vegetation on fires exists.

Laplace transforms	supplementary variable
stochastic process	Tasmanian rainforest

1. Introduction

In Tasmanian rainforest regions where the ecology has been predominantly unaffected by white man there are significant pockets of lower-grade vegetation (grass, shrub lands and eucalypt forests), in all covering half the area. This cannot be explained by soil type, climatic conditions or topological effects, as all favour rainforest species. Jackson [7] postulates that the dominant, if not the only, factor causing this vegetation anomaly is the occasional wild fire. These fires are completely destructive and level all vegetation, leaving in their path bare land with the occasional charred stump. Any fire of lesser intensity, that is, one that does not bodily destroy each tree, is not considered, as the effects described below are not applicable.

This paper demonstrates that fire/vegetation interactions and the present wild-fire frequencies lead to the distribution of forests encountered today. Further, this remarkable coexistence of species can only be

realised by a narrow band of fire frequencies – marginal increases or decreases cause major elimination of rainforests or lower-grade species respectively.

The model is developed using the supplementary variable technique [2,8]. While simpler approaches are possible – e.g., imbedded-chain analysis [6] – these do not lead to the variety of results developed here.

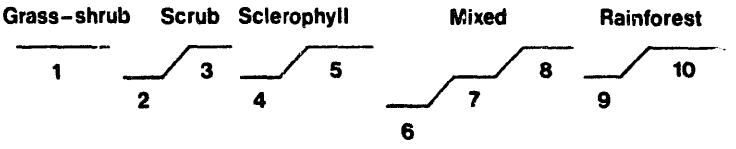
2. Interactions

Vegetation is categorised not by physical appearance but by the theoretical action of fire at any instant. As a consequence, distinction has to be made between "immature" and "mature" vegetation stands. Any forest bearing seeds is termed "mature", and a destructive fire in such a forest will act as a catalyst for the seeds, which germinate immediately and cause forest re-establishment. There is an initial growing period after the fire before this forest again produces seeds, and the vegetation at this time is termed "immature". During an immature period after a fire no regeneration of former vegetation occurs (there are no seeds present); instead, grasses soon cover the land, and these are quickly replaced by fire-resistant shrub species. There are four distinct forest types that display these reactions to fire: scrub and wet sclerophyll (both eucalypt forests), rainforest, and a sclerophyll/rainforest mixture. The structure of sclerophyll/rainforest (or "mixed" forest) causes another possibility following fire. Due to different growing times there is a period when only the eucalypts are mature. A fire at this time destroys the rainforest species but leaves eucalypt seeds to create a "pure" eucalypt forest. Grass and shrub species are fire resistant and regenerate after all fires.

Without fire a succession of vegetation types occurs until the dominant rainforest species prevail. Immature stands will be replaced by corresponding mature stands in the growing period of the forest species. A mature forest will be replaced by a more advanced mature vegetation in the order scrub, sclerophyll, mixed and rainforest. There is no possibility of ambiguity for the classification of a state at the time of succession – the first appearance of seeds of the succeeding vegetation is the point at which the nature, and hence the categorisation, of vegetation changes.

Of all species, rainforest saplings alone are able to survive their first years in a mature vegetation stand, and as a consequence rainforest can survive indefinitely without fires.

Table 1
Vegetation types with summary of effects of fire and fire free periods



(States 2, 4, 6, 7 and 9 are immature; 3, 5, 8, 10 mature)

Initial vegetation	Following fire	Following single succession
1	1	3
2	1	3
3	2	5
4	1	5
5	4	8
6	1	7
7	4	8
8	6	10
9	1	10
10	9	10

A detailed account of the effects of fires and reasons for the vegetation succession may be found in [5,7].

For the model, grasses and shrubs can be grouped together as one state. A fire in this state causes the regrowth of grasses followed by shrubs, while a fire-free period results in the establishment of scrub forest. The physical difference alone does not necessitate dual states.

The states and interactions can be summarised and numerically labelled as in Table 1.

The average time between fires depends on the vegetation cover; this time has been estimated to vary from 40 years for grass–shrubland to 300 years for rainforest. Fires are assumed to occur randomly in time. This is permissible as the magnitude of the periods between fire enables seasonal effects to ne neglected.

The succession of forest types in fire-free periods has a distribution which also depends on the vegetation. The transition from immature to corresponding mature forest occurs after the first seed deposit – the distribution has low variance. Transitions between mature forests occur as the initial forest thins due to death of trees, and by random introduction of higher-grade species. This gives a high-variance succession-time distribution.

3. Derivation of equations

The vegetation states will be labelled by numbers in Table 1.

Fires occur in a Poisson stream with parameter λ_i, $i = 1, 2, ..., 10$. The average time between fires for vegetation type i is $1/\lambda_i$.

The period required for succession of a forest will be a random variable with distribution function $G_i(x)$, where x is measured in years from the point of establishment of vegetation type i. At this stage the function $G_i(x)$ need not be specified further, as the model carries through using a general function. The form of $G_i(x)$ will be described when the model is applied to available data.

Define $\mu_i(x) = g_i(x)/(1 - G_i(x))$ to be the density that succession occurs at x conditional on no succession to time x ($g_i(x) = dG_i(x)/dx$).

For each state i define the joint probability density $P_i(x, t)$ as

$$P_i(x, t)\,dx = \mathbf{P}\ [\text{at time } t \text{ the vegetation is of type } i \text{ and of age between } x \text{ and } x + dx]\ .$$

Difference equations enable us to determine relations within and between these 10 states:

$$P_i(x+\Delta, t+\Delta) = P_i(x, t)\,(1 - \lambda_i\Delta + o(\Delta))\,(1 - \mu_i(x)\Delta + o(\Delta))$$

for an arbitrarily small period of time Δ. Taking limits on Δ we obtain

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right) P_i(x, t) = -P_i(x, t)\,(\lambda_i + \mu_i(x))\ . \tag{3.1}$$

The paths between states give the boundary equations

$$P_1(0, t) = \sum_{j=1,2,4,6,9} \lambda_j \int_0^\infty P_j(x, t)\,dx\ , \tag{3.2.1}$$

$$P_i(0, t) = \lambda_{i+1} \int_0^\infty P_{i+1}(x, t)\,dx\ , \quad i = 2, 9\ , \tag{3.2.2}$$

$$P_i(0, t) = \sum_{j=i-2}^{i-1} \int_0^\infty \mu_j(x)\,P_j(x, t)\,dx\ , \quad i = 3, 5, 10\ , \tag{3.2.3}$$

$$P_4(0, t) = \sum_{j=5,7} \lambda_j \int_0^\infty P_j(x, t)\,dx\ , \tag{3.2.4}$$

$$P_6(0, t) = \lambda_8 \int_0^\infty P_8(x, t)\, dx\,, \tag{3.2.5}$$

$$P_7(0, t) = \int_0^\infty \mu_6(x) P_6(x, t)\, dx\,, \tag{3.2.6}$$

$$P_8(0, t) = \sum_{j=5,7} \int_0^\infty \mu_j(x) P_j(x, t)\, dx\,. \tag{3.2.7}$$

Also, the normalising equation

$$\sum_{i=1}^{10} \int_0^\infty P_i(x, t)\, dx = 1\,, \tag{3.3}$$

must be satisfied.

4. Time-independent solution

In a steady-state regime $P_i(x, t) \to P_i(x)$, or alternatively $\partial P_i(x, t)/\partial t \to 0$. In this situation eq. (3.1) becomes

$$dP_i(x)/dx = -P_i(x)\,(\lambda_i + \mu_i(x))\,,$$

with solution

$$P_i(x) = P_i(0)\, \exp\{-\lambda_i x - \int_0^x \mu_i(y)\, dy\}$$

or

$$P_i(x) = P_i(0)\, \exp\{-\lambda_i x\}(1 - G_i(x))\,. \tag{4.1}$$

Also, the boundary and normalising equations, using result (4.1), reduce to a system of linear equations in the variables $P_i(0)$. Define the Laplace transform $\hat{G}_i(s)$ of $G_i(x)$ by

$$\hat{G}_i(s) = \int_0^\infty G_i(x)\, \exp\{-sx\}\, dx\,.$$

Then

$$P_1(0) = \sum_{j=1,2,4,6,9} P_j(0)\,(1 - \lambda_j \hat{G}_j(\lambda_j))\,, \tag{4.2.1}$$

$$P_i(0) = P_{i+1}(0)\,(1 - \lambda_{i+1}\,\hat{G}_{i+1}(\lambda_{i+1}))\,, \quad i = 2, 9\,, \tag{4.2.2}$$

$$P_i(0) = \sum_{j=i-2}^{i-1} P_j(0)\,\lambda_j\,\hat{G}_j(\lambda_j)\,, \quad j = 3, 5, 10\,, \tag{4.2.3}$$

$$P_4(0) = \sum_{j=5,7} P_j(0)\,(1 - \lambda_j\,\hat{G}_j(\lambda_j))\,, \tag{4.2.4}$$

$$P_6(0) = P_8(0)\,(1 - \lambda_8\,\hat{G}_8(\lambda_8))\,, \tag{4.2.5}$$

$$P_7(0) = P_6(0)\,\lambda_6\,\hat{G}_6(\lambda_6)\,, \tag{4.2.6}$$

$$P_8(0) = \sum_{j=5,7} P_j(0)\,\lambda_j\,\hat{G}_j(\lambda_j)\,, \tag{4.2.7}$$

and

$$\sum_{i=1}^{10} P_i(0)\,(1/\lambda_i - \hat{G}_i(\lambda_i)) = 1\,. \tag{4.3}$$

Eq. (4.2) are dependent; the solution is obtained by substituting the normalising equation (4.3) for one boundary equation. The relation

$$P_i(0)\,(1/\lambda_i - \hat{G}_i(\lambda_i)) = \int_0^\infty P_i(x)\,dx\,,$$

gives the probability that in steady state, the process having parameters λ_i and $\mu_i(x)$, the vegetation type will be i. As the forest region under consideration consists of a large number of independently acting areas, this value will be equal to the proportion of land carrying vegetation type i.

We have been unable to obtain any estimates other than Jackson's for the parameters used in this model. Jackson [7, p. 13] graphs a time scale

for fires and succession, which have been used to obtain

$$\lambda' = \begin{bmatrix} \lambda'_1 \\ \lambda'_2 \\ \vdots \\ \lambda'_{10} \end{bmatrix} = \begin{bmatrix} 0.025 \\ 0.025 \\ 0.0167 \\ 0.025 \\ 0.0111 \\ 0.02 \\ 0.0111 \\ 0.005 \\ 0.0111 \\ 0.0033 \end{bmatrix} = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_{10} \end{bmatrix} = \begin{bmatrix} 100 \\ 20 \\ 120 \\ 20 \\ 150 \\ 20 \\ 20 \\ 250 \\ 40 \\ \infty \end{bmatrix},$$

where l_i is the average succession time ($\int_0^\infty x\, g_i(x)\, dx$), and λ is a function of the number of destructive fires, independent of the total number of fires. For grass–shrub land, all fires clear vegetation, but the destructive burns through woody vegetation are often (depending on the type of vegetation) in the minority. For example, scrub fires occur every few years, but the conditions (favourable temperature, wind and humidity, as well as tinder dry growth) for a destructive fire to establish are met on average only every sixty years; in contrast, the conditions needed for a fire to establish in rainforest are usually sufficient for intensities to soar, resulting in most fires being severe.

Fires of low intensity have little or no long term effect on the vegetation distribution. Such fires clear fuel from the forest floor and reduce for a short time the probability of occurrence of further fires. However, an analysis of this phenomenon has shown that this has a negligible effect; and inclusion of this process complicates the model to the extent that the basic interactions are masked.

No information is available to the contrary that the rate of severe fires (due partly to the vegetation fire potential, but mainly to prevailing weather conditions) depends on the age of the forest, except for the change at maturity, and with the minimal effects of random low-intensity fires the assumption of uniform Poisson occurrence of intense fires appears valid for this region. Observations made in other regions add weight to this assumption [3].

The estimate of fire frequencies λ_i is the least accurate. The estimate arises from observations of the times between fires ($1/\lambda_i$). The occurrences of fires during this century are known, but this is too short a

term for useful estimates to be made. Alternatively, the distribution of the time to the last fire for each vegetation type may be calculated. For fires occurring in a Poisson stream in an enviornment of no succession this is an unbiassed estimate of the average time between fires. As succession occurs, the parameters must be weighted accordingly. It is assumed that any error in this estimate of the fire frequency is spread uniformly over all states, and the results are taken with $\lambda = k\lambda'$ for arbitrary values of k.

Two extreme choices of the distribution function $G_i(x)$ are used. The observations suggest that these functions should have low variance for immature and high variance for mature states. The functions chosen are:

(a) $G_i(x) = U(x - l_i)$, $i = 1, 2, ..., 10$, where $U(x)$ is the unit step function with jump at $x = 0$;

(b)

$$G_i(x) = \begin{cases} \int_0^x (4/l_i)^4 \, y^3 \exp\{-4y/l_i\} \, dy, & i = 1, 3, 5, 8, \\ \int_0^x U(y - \frac{4}{5}l_i) \, (20/l_i)^4 \, (y - \frac{4}{5}l_i) \\ \quad \times (1/3!) \exp\{-20(y - \frac{4}{5}l_i)/l_i\} \, dy, & i = 2, 4, 7, 9, \\ U(x - l_i), & i = 6. \end{cases}$$

Unlike the other states, the time points in state 7 are measured from the beginning of state 6. Although it means little difference to the results, for model accuracy state 6 is given constant length. The time points in state 7 are thus pseudo-independent of all other states.

Erlang functions are chosen for (b) only for ease of application of Laplace transforms.

The functions in (a) have variances which are lower and those in (b) higher than the natural process. Table 2 compares the results using these functions for values $\lambda = k\lambda'$, $k = 0.3, 0.6, 0.9$. There is only a marginal difference between the results due to different succession functions, certainly close to $\lambda = 0.6\lambda'$. This value, because the results correspond to observation, can be described as the feasible region. The similarity of the results can be expected because the only significant variables determining the vegetation distribution are the fire frequency and the mean succession time.

Table 2
Comparison of steady-state results for (a) unit step and (b) Erlang vegetation succession distributions

State	$0.3\lambda'$		$0.6\lambda'$		$0.9\lambda'$	
	(a)	(b)	(a)	(b)	(a)	(b)
1	0.020	0.018	0.162	0.122	0.515	0.358
2	0.002	0.002	0.101	0.016	0.039	0.040
3	0.021	0.018	0.101	0.087	0.163	0.165
4	0.001	0.002	0.010	0.010	0.014	0.019
5	0.023	0.023	0.081	0.083	0.077	0.110
6	0.001	0.001	0.005	0.004	0.004	0.006
7	0.001	0.003	0.004	0.010	0.003	0.012
8	0.033	0.032	0.087	0.084	0.054	0.074
9	0.032	0.032	0.035	0.038	0.012	0.019
10	0.866	0.869	0.498	0.546	0.119	0.197

The unit step function gives results weighted towards the (numerically) lower states. These probabilities rapidly change to the figures quoted for function (b) upon the introduction of dispersion about the mean, this ensuring that (b) represents the more accurate estimate. In spite of this, as the differences are marginal in the feasible region, future results are given using only step functions (a); this leads to a considerable simplification when we solve time-dependent equations numerically.

Fig. 1 gives steady-state results when the parameter k is varied continuously. At $\lambda = 0.6\lambda'$, the vegetation proportions are

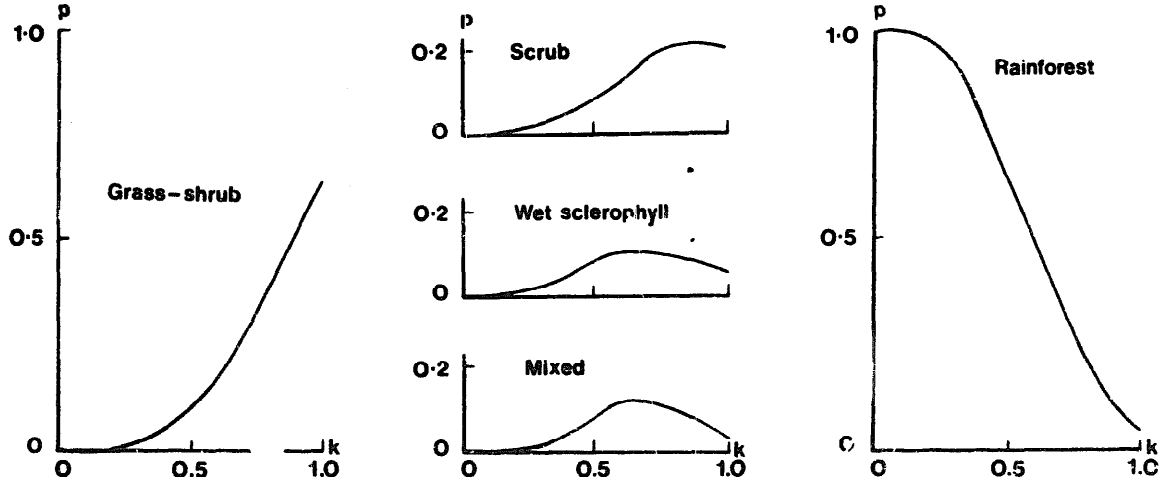


Fig. 1. Steady-state domain results: The porportion p of time the land carries each vegetation cover, when $G_i(x) = U(x - l_i)$ and $\lambda = k\lambda'$.

grass–shrub 0.16,
scrub 0.12,
wet schlerophyll 0.19,
mixed forest 0.10,
rainforest 0.53,

with a proportion 0.08 of the vegetation immature. This is consistent with the present observed levels.

Given a fire frequency $\frac{3}{5}$ of that estimated, and with the problems of estimation this can be readily accepted, fire alone would result in half the region not supporting rainforest. Further the proportions of non-rainforest vegetation are in close agreement with the present levels. Thus the occurrence of wild fires can in the long term cause at least partly today's anomalous situation. If other factors not considered also add to this effect, then the fire frequency should be less than $0.6\lambda'$ – the estimate λ' would be smaller. This supports at least superficially that fires are the major, and possibly the only, cause of this situation.

From fig. 1 it is observed that at the value $\lambda = 0.6\lambda'$ the vegetation distribution is unstable. A change of 10% of the fire frequency gives a long term change of 20% of the rainforest proportion. As these results give no time scale for this change, a time-dependent evaluation has to be made.

5. Time-dependent solutions

Define the Laplace transform of $P_i(x, t)$ by

$$\hat{P}_i(x, s) = \int_0^\infty P_i(x, t) \exp\{-st\}\, dt\,, \quad \mathrm{Re}\, s > 0\,.$$

On taking the Laplace transforms of (3.1) we obtain

$$\frac{\partial \hat{P}_i(x, s)}{\partial x} + \hat{P}_i(x, s)\,(s + \lambda_i + \mu_i(x)) = P_i(x, 0)\,.$$

Integrating with respect to x gives

$$\hat{P}_i(x, s) = \exp\{-(s + \lambda_i)x\}(1 - G_i(x)) \times$$

$$\times \left[\int_0^x P_i(z, 0) \exp\left\{\int_0^z (s + \lambda_i + \mu_i(y))\mathrm{d}y\right\} \mathrm{d}z + \hat{P}_i(0, s)\right] \tag{5.1}$$

When (5.1) is substituted into the Laplace transforms of the boundary equations (3.2), a system of linear equations in $\hat{P}_i(0, s)$, but with unknown functions $P_i(z, 0)$, is obtained. It is advantageous to consider the initial conditions $P_i(z, 0)$ before this substitution is performed. Let the process be in a steady-state regime with parameters λ^* which change instantly to λ. Then

$$P_i(z, 0) = P_i(0, 0) \exp\{-\lambda_i^* z\} \, (1 - G_i(z)) \, , \quad i = 1, 2, \ldots, 10 \, . \tag{5.2.1}$$

On substitution in (5.1) we obtain

$$\hat{P}_i(x, s) = \exp\{-\lambda_i x\} \, (1 - G_i(x))$$

$$\frac{P_i(0, 0) \exp\{(\lambda_i - \lambda_i^*)x\}}{s + \lambda_i - \lambda_i^*} \, [1 - \exp\{-(s + \lambda_i - \lambda_i^*) \, x\}$$

$$+ \exp\{-sx\} \, \hat{P}_i(0, s)] \, . \tag{5.22}$$

Hence

$$\int_0^\infty \hat{P}_i(x, s) \, dx = A_i(s) + B_i(s) \, \hat{P}_i(0, s) \, , \tag{5.3.1}$$

$$\int_0^\infty \hat{P}_i(x, s) \, \mu_i(x) \, dx = C_i(s) + D_i(s) \, \hat{P}_i(0, s) \, , \tag{5.3.2}$$

where

$$A_i(s) = \frac{P_i(0, 0)}{s + \lambda_i - \lambda_i^*} \left\{ \frac{1}{\lambda_i^*} - \frac{1}{s + \lambda_i} - \hat{G}_i(\lambda_i^*) + \hat{G}_i(s + \lambda_i) \right\} \, , \tag{5.3.3}$$

$$C_i(s) = \frac{P_i(0, 0)}{s + \lambda_i - \lambda_i^*} \{\lambda_i^* \hat{G}_i(\lambda_i^*) - (s + \lambda_i) \, \hat{G}_i(s + \lambda_i)\} \, . \tag{5.3.4}$$

$$B_i(s) = 1/(s + \lambda_i) - \hat{G}_i(s + \lambda_i) \, , \tag{5.3.5}$$

$$D_i(s) = (s + \lambda_i) \, \hat{G}_i(s + \lambda_i) \, . \tag{5.3.6}$$

On taking the Laplace transforms and substitution of (5.4.1) and (5.4.2), the system (3.2) can be written

$$P = R\hat{P} + S1 ,$$

where

$$\hat{P} = \begin{bmatrix} \hat{P}_1(0, s) \\ \hat{P}_2(0, s) \\ \vdots \\ \hat{P}_{10}(0, s) \end{bmatrix} , \qquad 1 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} ,$$

and where R and S are the appropriate 10×10 matrices as determined by the boundary equations. The normalising equation (3.3) is introduced via the initial conditions. The solution is

$$\hat{P} = -(R - I_{10})^{-1} S1 ,$$

where I_{10} in the 10×10 identity matrix.

The probability that the vegetation is of type i at time t (that is, $\int_0^\infty \hat{P}_i(x, t)\, dx$) can be calculated by taking the inverse Laplace transforms on the point values of

$$\int_0^\infty \hat{P}_i(x, s)\, dx = A_i(s) + B(s)\, \hat{P}_i(0, s) .$$

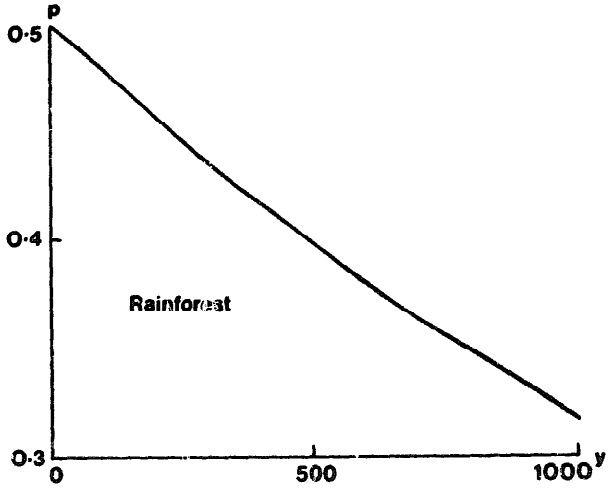


Fig. 2. The proportion p of rainforest at time y (in years) following a change of fire frequencies from $\lambda = 0.6\lambda'$ (steady-state proportion 0.5 rainforest) to $\lambda = 0.85\lambda'$ (0.14).

A convenient algorithm for this purpose is that of [4], where regularity conditions are satisfied.

Fig. 2 shows the slow expected change of rainforest vegetation when the fire frequency parameters are increased from $\lambda = 0.6\lambda'$ to $\lambda = 0.85\lambda'$ and when the succession distribution functions $G_i(x)$ are unit step. The curve is not as smooth as indicated, the complex interactions involved causing minor fluctuations. A least-squares analysis from 100 points along this curve gives an extrapolation formula for use beyond $t = 1000$ years, viz.

$$\int_0^\infty P_{10}(x, t)\, dx = 0.375 \exp\{-6.44t \times 10^{-4}\} + 0.139 .$$

(The Laplace transform inversion loses accuracy with the distance from the zero point. An unreasonable number of transform points are required to obtain an accurate inversion beyond $t = 1000$ years. As an exponential decline in the rainforest proportion is expected, the extrapolation formula presents, for our present purposes, an excellent estimation of the long term behaviour.)

Using this extrapolation formula, a one third shift to a new steady-state regime requires a 600 year period, and a half shift of the order of 1700 years. The long-term change in rainforest proportions (or other vegetation types) is substantial, but long periods elapse before noticeable change will occur.

Related results from time-dependent equations

Returning to eq. (5.1), use as initial conditions the artificial requirement that all vegetation be initially of one type and of one age, i.e. choose

$$P_i(z, 0) = 0 , \quad \text{for all } i \neq i_0, z ,$$

$$P_i(z, 0) = \delta(z - 0) , \tag{5.4.1}$$

(where $\delta(z)$ is the Dirac delta function.) On substitution in (5.1), we obtain

$$\hat{P}_i(x, s) = \begin{cases} \exp\{-(\lambda_i + s)x\}\,(1 - G_i(x))\hat{P}_i(x, 0) , & i \neq i_0 , \\ \exp\{-(\lambda_i + s)x\}\,(1 - G_i(x))\,(1 - \hat{P}_i(x, 0)) . & \end{cases} \tag{5.4.2}$$

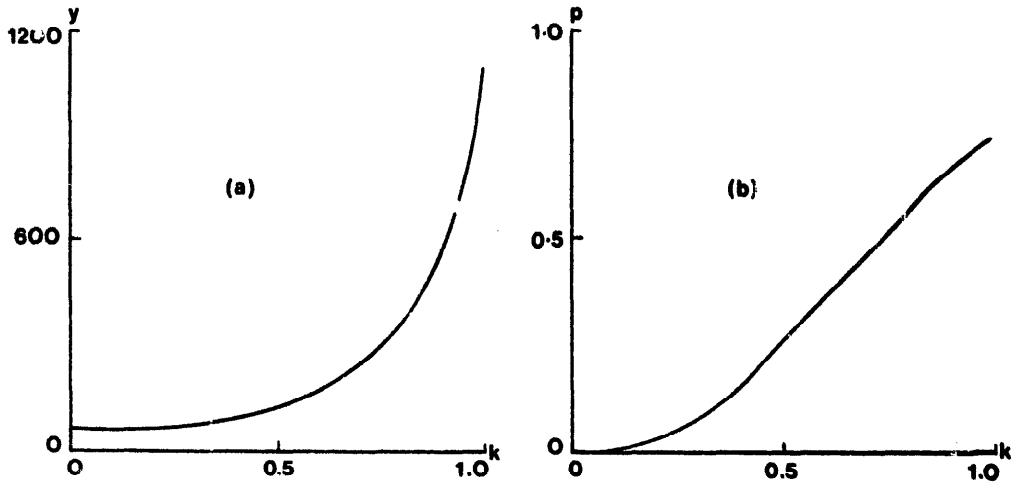


Fig. 3. (a) The average time y in years for the transition from states 1 to 10. (b) The probability of entering state 1 before 10, where initiallv in state 5 and of age zero. $G_i(x) = U(x - l_i)$ and $\lambda = k\lambda'$.

Further statistics of the process may now be obtained by inverting specific functions of $\hat{P}_i(x, s)$. Consider the following examples.

(A) With $i_0 = 1$ in (5.3.1), and with the artificial condition $\lambda_{10} = 0$, the average time required to first reach state 10 (mature rainforest) from state 1 (grass–shrub) (fig. 3(a)) can be calculated as

$$\int_0^\infty t\, P_{10}(0, t)\, \mathrm{d}t = \lim_{s \to 0} \int_0^\infty t\, P_{10}(0, t)\, \mathrm{e}^{-st}\, \mathrm{d}t$$

$$= \lim_{s \to 0} \left\{ - \frac{\mathrm{d}\hat{P}_{10}(0, s)}{\mathrm{d}s} \right\},$$

i.e., by requiring $\lambda_{10} = 0$ the rainforest is forced to an absorbing state; the result required is the average absorption time. A more efficient method of obtaining first-entrance times, however, is through a semi-Markov analysis [9].

(B) Let $i_0 = 5$, $\lambda_{10} = 0$ and $l_1 = \infty$ in (5.3.1). Both grass–shrub land and mature rainforest are absorbing, and from an initial state of mature sclerophyll forest we have

$$P \text{ (entering grass–shrub before rainforest)} =$$

$$= \lim_{t \to \infty} \int_0^\infty P_1(x, t)\, \mathrm{d}x$$

$$= \lim_{s \to 0} s \int_0^\infty \hat{P}_1(x, s)\, dx ,$$

P (entering rainforest before grass–shrub) =

$$= \lim_{s \to 0} s \int_0^\infty \hat{P}_{10}(x, s)\, dx .$$

These dual probabilities, for parameters as stated, are given in Figure 3(b).

6. Conclusion

The present model was proposed to examine the theory that, in the long term, fires by themselves have a significant effect on the vegetation horizon. It appears that even these rare wild fires (at $\lambda = 0.6\lambda'$ the average time between fires varies from 70 to 500 years depending on the vegetation) can cause the destruction of half the indigenous rainforest cover. No assumptions made – such as homogeneous fire frequencies, infinite independent regions, zero forest creep, the classification and interrelation of states – would appear to result in a significant error in the model; testing has demonstrated that it is the fire frequencies and average succession times alone that can have a marked effect with only small changes.

The results obtained also imply superficially that it is probable that no other factors cause a significant destruction of the rainforest. With fire as the only cause, the frequencies have been over-estimated by 67%. With other effects, the frequencies would necessarily have to be further over-estimated. Also, fire gives proportions of other vegetation types (at rainforest proportion 50% or slightly more) which are consistent with the observations. If other factors cause rainforest destruction, it could be expected that by setting parameters to realise 50% rainforest the percentage of other vegetation types would alter from observed levels.

If it can be accepted that fires have a significant effect on the forect distribution, any possible change with the advent of white man must be considered. Even small variations in frequencies have drastic long term effects, but, as indicated by the time-dependent results, such changes occur only slowly. Even when the assumption of "an infinite number" of independent sub-regions is replaced by "a large number" (in this si-

tuation fires will have a more marked short-term effect, but the status quo will be maintained after a fire-free 20 year period when any mature forest will return), a long period may elapse before a change occurs.

When the conditions of temperature, humidity and air movement are optimal for destructive fires, they are all but impossible to halt artificially. Likewise, as is the experience with man, prevention of fires will never be attained, so it may be that planting will have to be carried out to maintain the Tasmanian rainforests at their present level if white man, by his presence, has inadvertently raised the fire potential frequency.

It is quite possible, however, that this fire frequency has been reduced over the past two centuries as the former aboriginal inhabitants and their practices have disappeared. If this is the case, a slight increase in rainforest cover can be expected over the next few centuries.

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References

[1] R.E. Bellman et al., Numerical Inversion of the Laplace Transform (Elsevier, Amsterdam, 1966).
[2] D.R. Cox, The analysis of non-Markov stochastic processes by the inclusion of supplementary variables, Proc. Cambridge Philos. Soc. 51 (1955) 433–441.
[3] A.A. Cunningham and D.L. Martell, A stochastic model for the occurrence of man-caused forest fires, Can. J Forestry Res. 3 (1973) 282–287.
[4] H. Dubner and J. Abate, Numerical inversion of Laplace transforms by relating them to the finite Fourier cosine transform, J. Assoc. Comp. Mach. 15 (1968) 115–123.
[5] J.M. Gilbert, Forest succession in the Florentine Valley, Tasmania, Papers and Proc. Roy. Soc. Austral. 93 (1958) 129–151.
[6] W. Henderson and C.W. Wilkins, On the interaction of bushfires and vegetation, Search 6 (1975) 130–133.
[7] W.D. Jackson, Fire, air, water and earth – an elemental ecology of Tasmania, Proc. Ecol. Soc. Austral. 3 (1968) 9–16.
[8] S. Karlin, A first course in Stochastic Processes (Academic Press, New York, 1969).
[9] C.W. Wilkins, On first passage times of semi-Markov processes.